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Wave Drag of Optimum and Other Boat Tails

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1. Introduction

MINIMIZING the boat tail drag of an aircraft or a missile is a problem of considerable interest to the design engineer. Two particular aspects of the problem are considered in this Note. First, the shapes of minimum-drag boat tails were determined for supersonic flow. Secondly, the wave drag coefficients of these optimum bodies were compared to the drag coefficients of other commonly used boat tail contours such as conical, circular arc and parabolic arc.

2. Minimum-Drag Boat Tails

The problem of optimizing the boat tail shape can be posed as illustrated in Fig. 1. Given the length and radii of the boat tail and the ambient Mach number, what contour connecting points A and B will result in the least wave drag?

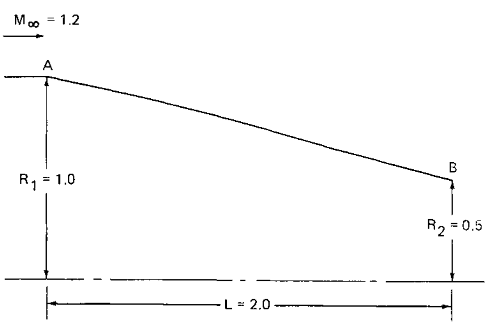

Fig. 1 Optimum boat tail contour for $L/R_1 = 2.0$, $R_2/R_1 = 0.5$, and $M_\infty = 1.2$.

Received March 4, 1970. The author wishes to acknowledge the assistance of H. Burk in running the many cases with the method of characteristics program and in organizing the final results.

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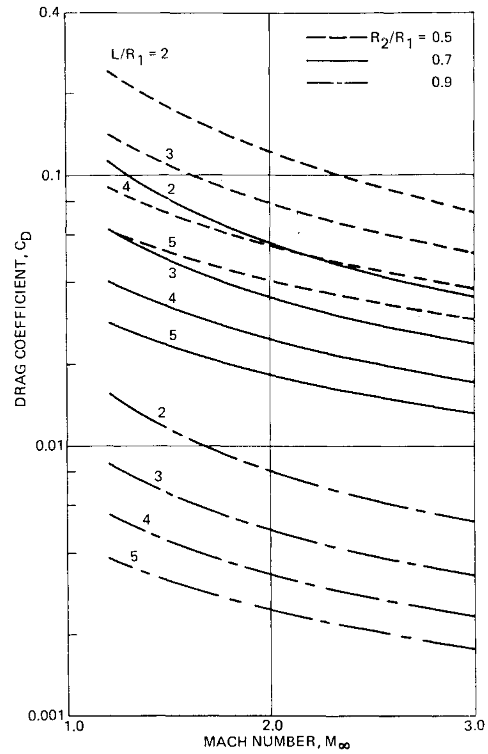


Fig. 2 Drag coefficients of optimum boat tails.

The appropriate way to analyze problems of this type is by way of calculus of variations. In searching the literature, it was discovered that although the problem had not been treated exactly, it had been solved for linearized supersonic flow. (No attempt at optimization using the full nonlinear equations was made within the scope of the present effort). The linearized analysis of Parker[1] considered the problem of minimizing the transition section drag connecting two axisymmetric cylinders with the larger one located downstream. Since linearized equations were used, the flow direction can be reversed and the solution would remain unchanged to the first order. Thus, the results of Parker's analysis are directly applicable to the boat tail problem. Parker does not arrive at an explicit relationship $R = f(x)$, but rather derives an integral equation relating R and x

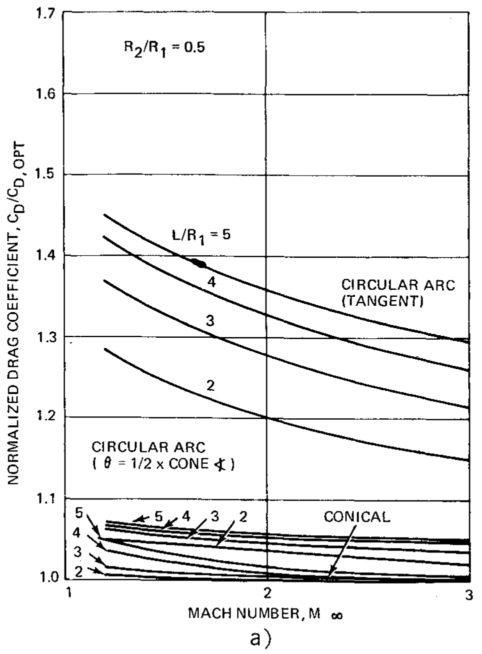
$$\beta^2(R^2 - R_2^2) = \frac{4\beta^2(R_1^2 - R_2^2)(1 + \beta R_1 + \beta R_2)}{\pi(1 - \beta R_1 + \beta R_2\ (1 + 3\beta R_1 + \beta R_2)} \times \int_\pi^{\cos^{-1}\left[\frac{2(x - \beta R) - (1 + \beta R_1 - \beta R_2)}{1 + \beta R_1 + \beta R_2}\right]} \cos\theta \left\{\left[x - \frac{1 + \beta R_1 - \beta R_2}{2} - \frac{(1 + \beta R_1 + \beta R_2)}{2}\cos\theta\right]^2 - \beta^2 R^2\right\}^{1/2} d\theta \quad (1)$$

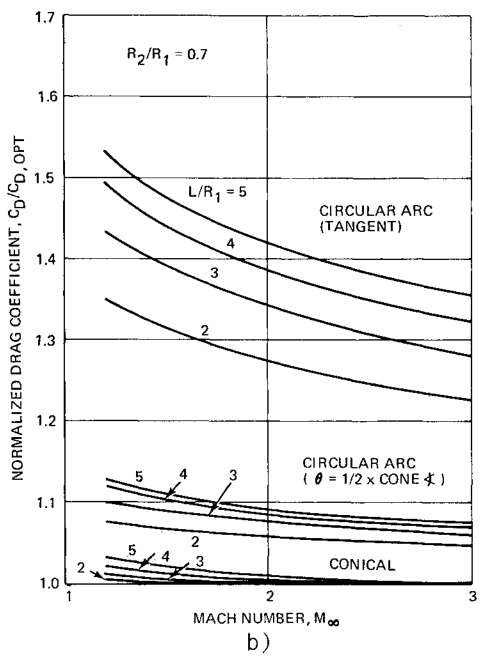
where

$$\beta = (M_\infty^2 - 1)^{1/2} \quad (2)$$

A computer program was written to solve this equation numerically. Thus, for given values of R_1, R_2, L, and M_∞ the program could generate a table of R values corresponding to a selected range of x values.

The previous program was used to generate minimum drag profiles for boat tails having thickness and length ratios of practical interest. Thickness ratios (R_2/R_1) of 0.5, 0.7, and 0.9 and length ratios (L/R_1) of 2, 3, 4, and 5 were selected for these computations. The freestream Mach number was taken equal to 1.2, 2.0, and 3.0. It was observed that, although the computed contours were curved and exhibited an inflection point, they were all rather close to conical in





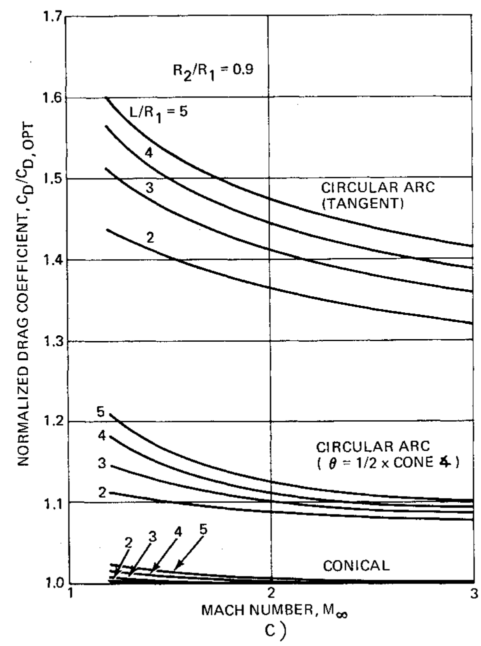


Fig. 3 Boat tail wave drag normalized by drag of optimum configuration: a) $R_2/R_1 = 0.5$; b) $R_2/R_1 = 0.7$; c) $R_2/R_1 = 0.9$.

shape for the range of values considered in this study. This was particularly evident for short boat tails and for higher Mach numbers. An example of these results is shown in Fig. 1. The remaining contours are not presented since it would be difficult to distinguish graphically between the various shapes. These contours were, however, made use of in the analysis described below.

3. Comparison of Drag Coefficients

It was considered of interest to compare the drag coefficients of the optimum boat tails to the drag coefficients of other commonly used shapes. To perform this comparison, a method of characteristics computer program was used to evaluate the drag coefficients of the various boat tails. In addition to the optimum boat tails, the other geometries considered were 1) conical, 2) circular arc tangent at the shoulder, 3) circular arc intersecting the cylindrical forebody at an angle equal to $\frac{1}{2}$ of the corresponding cone angle, 4) parabola with the apex tangent at the shoulder, and 5) parabola intersecting the cylindrical forebody at an angle equal to $\frac{1}{2}$ of the corresponding cone angle.

The computed drag coefficients (based on the area πR_1^2) of the optimum boat tails are summarized in Fig. 2. Since these drag coefficients represent the theoretical minimum (except for slight errors introduced by the assumption of linearized flow in Parker's analysis), they were used as a reference in comparing drag coefficients of all the other shapes. The resulting normalized drag coefficients of the conical and two different circular arc boat tails are shown in Fig. 3. Because the drag coefficients of parabolic arc boat tails were found to be very close to those of corresponding circular arc boat tails (maximum difference <2%), they were not included in the figure. The three graphs (a, b, and c) in Fig. 3 correspond to boat tail radii ratios of 0.5, 0.7, and 0.9. The effect of boat tail length ratio on $C_D/C_{D,\text{opt}}$ is shown parametrically for the different configurations.

A number of conclusions can be drawn from the results in Fig. 3. For the boat tails considered, the drag coefficient becomes relatively closer to the optimum value as the Mach number increases or the boat tail length ratio decreases. The most important conclusion, however, is that the wave drag of boat tails formed by a circular arc tangent at the shoulder is substantially higher than the drag of optimum boat tails. A lower drag is observed when the circular arc intersects the forebody on an angle equal to $\frac{1}{2}$ the cone angle. The lowest drag is noted for the conical boat tails. In fact, for the range of conditions covered in this study, the cone drag was never more than 5% higher than the drag of the optimum boat tail. Thus, in supersonic flow, a conical boat tail comes very close to having a minimum wave drag. Our studies have neglected the displacement effects of the boundary layer. It is expected that, although the viscous effects will change to some extent the quantitative results, the general trends observed would still be valid in a real flow situation. These findings are corroborated in part by some recent experiments (Ref. 2) in which four different boat tail shapes were tested in a wind tunnel (at $M = 2.5$, 3, and 3.5) and the conical boat tail clearly indicated the lowest drag.

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